J.D. Jackson Problem 5.6

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To begin I'll find the B-field due to a cylindrical conductor of radius a that does *not* have a hole bored in it.

$$\oint \mathbf{B_a} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_{enc} \tag{1}$$

We know by symmetry that the B-field would be equal in magnitude, so it can come out of the integral, and we obtain a familiar result.

$$B_a = \frac{\mu_0}{2} jr \tag{2}$$

In full vector notation, we use the right hand rule to obtain

$$\mathbf{B_a} = \frac{\mu_0}{2}(\mathbf{j} \times \mathbf{r}) \tag{3}$$

By the same logic we see that the B-field from *only* a conducting cylinder of radius b a distance d from the origin is

$$\mathbf{B_b} = \frac{\mu_0}{2}(\mathbf{j} \times (\mathbf{r} - \mathbf{d})) \tag{4}$$

Now the critical step is that the actual B-field in the hole is given by

$$\mathbf{B_{tot}} = \mathbf{B_a} - \mathbf{B_b} \tag{5}$$

$$= \frac{\mu_0}{2}[\mathbf{j} \times \mathbf{r} - \mathbf{j} \times (\mathbf{r} - \mathbf{d})] \tag{6}$$

$$= \frac{\mu_0}{2}\mathbf{j} \times [\mathbf{r} - \mathbf{r} + \mathbf{d}] \tag{7}$$

$$\boxed{\mathbf{B_{tot}} = \frac{\mu_0}{2}(\mathbf{j} \times \mathbf{d})} \tag{8}$$